Statistical learning generates implicit conjunctive predictions

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Abstract

The cognitive system readily detects statistical relationships where the presence of an object predicts a specific outcome. What is less known is how the mind generates predictions when multiple objects predicting different outcomes are present simultaneously. Here we examine the rules with which predictions are made in the presence of two objects that are associated with two distinct outcomes. In three experiments, participants first implicitly learned that an object predicted a specific target location in a visual search task. When two objects predicting two different target locations were present simultaneously, participants were reliably faster to find the target when it appeared in the conjunctive location than in disjunctive locations. This was true even if participants were not consciously aware of the association between the objects and target locations. The results suggest that in the presence of multiple predictors, statistical learning generates implicit expectations about the outcomes in a conjunctive fashion.

Keywords: Implicit learning, regularities, conjunctive inference, visual search, attention

Introduction

The visual environment contains widespread regularities in terms of co-occurrences between individual objects or events over time. For example, the red light turns on after the yellow light at traffic intersections, and thunder follows lightening in a thunderstorm. The mind can detect such regularities effortlessly, automatically, or even outside of conscious awareness. One form of extracting these regularities (i.e., A predicts B) is statistical learning, which involves the detection of statistical relationships among individual objects over space or time (Fiser & Aslin, 2001; Saffran, Aslin, & Newport, 1996; Turk-Browne, Jungé, & Scholl, 2005). Statistical learning occurs incidentally to ongoing tasks and quickly after a few exposures to the regularities (Turk-Browne, Scholl, Johnson, & Chun, 2010), and proceeds without explicit awareness or conscious intent (Baker, Olson, & Behrmann, 2004).

The implicit extraction of regularities has a number of consequences on the representations of the individual objects that comprise the regularities. Recent studies suggest that statistical learning spontaneously biases attention to the co-occurring objects in a persistent manner (Zhao, Al-Aidroos, & Turk-Browne, 2013; Yu & Zhao, 2015), interferes with summary perception (Hall, Mattingley, & Dux, 2015; Zhao, Ngo, McKendrick, & Turk-Browne, 2011), updates object representations (Yu & Zhao, 2018a, Yu & Zhao, 2018b), facilitates the compression of information in working memory (Brady, Konkle, & Alvarez, 2009; Zhao & Yu, 2016), and leads to automatic transitive inferences (Luo & Zhao, 2018).

To date, research on statistical learning has predominately focused on the relationship between individual objects or events. However, in the broader visual environment different objects or events are often present at the same time where each predicts a specific outcome. For example, excessive smoking can lead to cardiovascular problems as well as lung complications, while excessive alcohol consumption can lead to similar cardiovascular problems and also potential brain damage. When excessive smoking occurs with excessive drinking, what consequences would follow? In this example, a conjunctive inference would generate an expectation that satisfies both predictors (i.e., cardiovascular problems), whereas a disjunctive inference would generate an expectation that satisfies either one of the two predictors (i.e., cardiovascular problems, lung complications, and potential brain damage). When people are presented with both predictors at the same time, what kind of inference do they make automatically (Mendelson, 2009)? Understanding automatic conjunctive or disjunctive inferences can help illuminate reasoning biases such as the conjunction fallacy where people mistakenly judge a conjunctive statement to be more probable than a disjunctive statement (Tversky & Kahneman, 1983).

In the current study, we examine the rules with which predictions are made in the presence of two objects that are associated with two distinct outcomes. In a visual search paradigm, participants first viewed one color circle and then searched for a target (a rotated T) in an array during the exposure phase. Each color predicted a specific location of the target in the array. For example, after a blue circle the target would always appear in the top half of the array; and after a red circle the target would always appear in the left half of the array. The question is: Where was the target expected to appear when both the blue circle and the red circle were present at the same time? A conjunctive prediction would suggest that the target was expected to appear in the top left quadrant of the array, whereas a disjunctive prediction would suggest that the target was expected to appear in the top half or the left half of the array. Importantly, at the inference phase when both color circles were present, the target was equally likely to appear in any quadrant of the array. We used response time of target search during the inference phase to gauge in which location the target was expected to appear.

In Experiments 1 and 2, we found that participants were reliably faster to find the target when it appeared in the conjunctive quadrant than in the disjunctive quadrant. This was true even if participants were not consciously aware of the association between the color circles and target locations during debriefing. We further replicated the finding in Experiment 3 where the two predictors were two feature dimensions in one object. This effect was equally strong whether participants implicitly learned the association or were explicitly told about the association.

Experiment 1

This experiment examined which type of inference participants would make when they saw a pair of colors, each predicting a different half of the array.

Participants

A total of 120 students (81 female, mean age=20.0 years, SD=2.3) from the University of British Columbia (UBC) participated for course credit.

Stimuli

For each trial in the experiment, participants saw one colored circle first, followed by a search array (Figure 1). The color circle could appear in one of four colors (R/G/B): red (255/0/0), yellow (255/255/0), blue (0/0/255), or grey (192/192/192). Each circle subtended 2.2° of visual angle. For each search array following the circle, 16 objects were presented in an invisible 8-by-8 grid. Each cell in the grid subtended 1.7° of visual angle. The 8-by-8 grid was divided into four 4-by-4 quadrants, where each quadrant was separated from the adjacent two quadrants by 2.2° of visual angle. Each quadrant contained four objects, where no row or column in the quadrant could be empty.

Out the 16 objects in each array, 15 were distractors in “L” shapes, randomly pointing to the left or right. There was only one target in each array, which was a rotated “T”, randomly determined to be pointed to the left or right. Participants were asked to find the target “T” and indicate which direction the “T” was pointing (left or right) by pressing a key on the keyboard, as quickly and accurately as possible.

For each trial, the color circle was presented on the screen for 1000ms. Followed by a 1000ms blank screen, the search array appeared on the screen until response. There was a 1000ms blank screen interval between trials.

Procedure

Participants first completed the exposure phase (Figure 1). During exposure, one color circle appeared on the screen at a time followed by a visual search array. Each of the four colors was presented for 40 times during exposure, resulting in a total of 160 trials (the order of the trials was random). Each color predicted that the target “T” in the search array always appeared in a unique half of the array (the top, left, bottom, or right half). For example, after the blue circle, the target always appeared in the top half of the array. After the red circle, the target always appeared in the left half of the array. The target location within each half of the array was counter-balanced between the two quadrants (e.g., counterbalanced between top-left and top-right quadrants for the top half of the array), and the target location within each quadrant was randomly determined. The color-location associations were randomly determined for each participant but remained fixed throughout the experiment for the participant.

We wanted to examine whether there were differences in conjunctive inferences made from explicit knowledge versus incidentally learned predictions. Therefore, half of the participants (N=60) were randomly selected to be explicitly told about the associations between colors and target locations before exposure (explicit condition), and the other half were told to only pay attention to the color circle and search for the target (implicit condition).

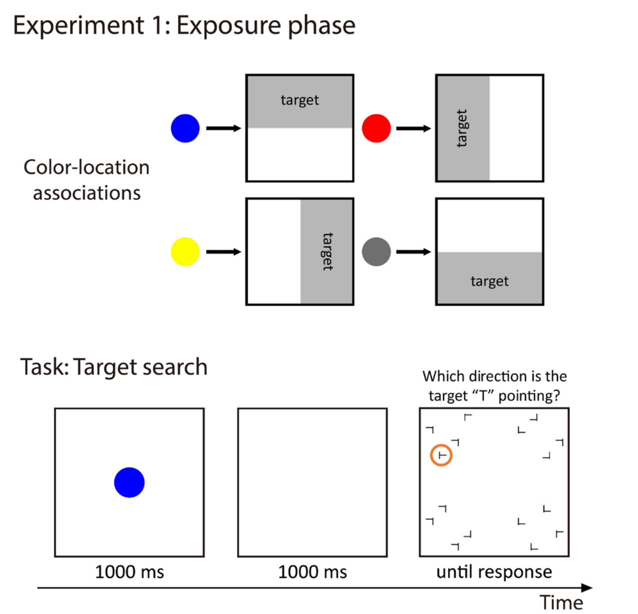

Figure 1. Experiment 1 exposure phase. Each color circle predicted the location of the target in the subsequent search array. In the visual search task, participants saw the color circle first, and then searched for a target (the rotated “T”) and judged the direction of target as quickly and accurately as possible.

After exposure, participants completed the inference phase (Figure 2). During this phase, two color circles were presented at the same time in each trial, followed by a search array. There were six unique color pairs. Each color pair and the following search array were presented four times in the inference phase in a random order, resulting in 24 trials in total. In each trial, the target appeared in any of the four quadrants with equal probability (the top-left, top-right, bottom-left, and bottom right quadrant). The location of the target within the quadrant was randomly determined.

Since the target now appeared in the four quadrants with equal probability, faster response time in target search in a given quadrant would indicate that the participant prioritized that quadrant for target search. This would mean that the participant expected that the target would appear in that quadrant, suggesting a prediction of where the target would appear after seeing the two color circles.

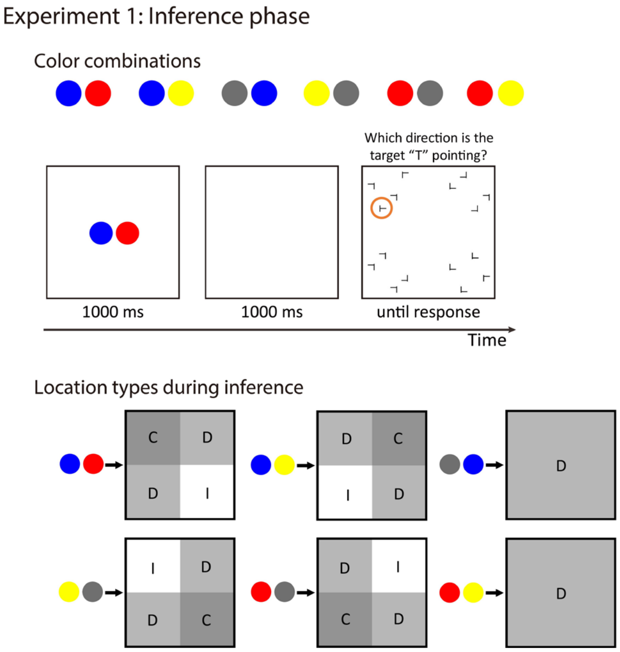

Figure 2. Experiment 1 inference phase. The four colors were combined into six color pairs. The pairs were presented first, followed by a search array. The target appeared in all four quadrants with equal probability following each pair. Based on the color-location associations during exposure, there were four types of target location following each color pair. These include the locations consistent with a conjunctive inference (C), locations consistent with a disjunctive inference (D), and the impossible locations (I).

During the inference phase, the two color circles were presented next to each other horizontally or vertically (randomly determined), and the order of the two colors for each pair was counter-balanced. Based on the color-location associations during exposure, there were four types of target location following each pair: locations consistent with a conjunctive inference (C), locations consistent with a disjunctive inference (D), and the impossible locations where the target would never appear based on the prior color-location associations (I). In both the explicit and implicit conditions, participants were only told that they would now see two color circles appearing simultaneously on the screen before each search array, and they were asked to search for the target as in the exposure phase.

After the inference phase, participants in the implicit condition also completed a test phase to probe their awareness of the color-location associations. They were asked where the target would appear (the top, left, bottom, and right half of the array) after seeing each of the four colors, so guessing would result in an accuracy of 0.25 in the test phase.

Results and Discussion

The test phase accuracy for participants in the implicit condition was 0.51, reliably above chance [chance=0.25, p<.001], indicating that participants in the implicit condition have successfully learned the color-location associations.

We then analyzed the responses time (RT) of correct trials in the inference phase to see what type of inferences participants made when they saw the color pairs. We grouped the trials in the inference phase into four types: conjunction, disjunction (2 quadrants vs. 4 quadrants), and impossible. Take the blue and red pair, the blue circle previously predicted that the target would appear in the top half of the array and the red circle previously predicted that the target would appear in the left half of the array. This means that the top left quadrant was the conjunctive quadrant, the top right and the bottom left quadrants were the disjunctive quadrants, and the bottom right quadrant was the impossible quadrant. For example, faster RT in the conjunctive quadrant would indicate that participants expected the target would appear in that quadrant, suggesting a conjunctive prediction. We plotted the RT in each type of quadrant in the inference phase (Figure 3).

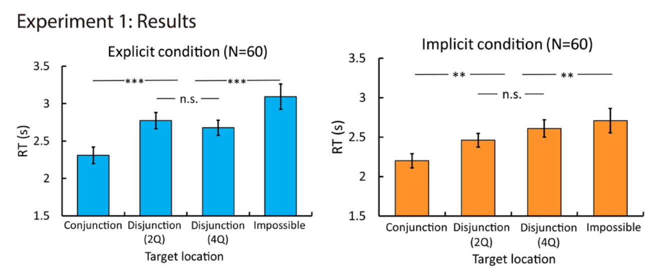

Figure 3: Experiment 1 results. The response time (RT) for each type of trials was graphed separately for the explicit and implicit conditions (Error bar reflect ± 1 SE; **p<.01, ***p<.001).

A 2 (condition: explicit vs. implicit, between-subjects) × 4 (trial type: conjunctive, 2-quadrant disjunctive, 4-quadrant disjunctive, and impossible quadrant, within-subjects) mixed-design ANOVA revealed a significant main effect of trial type [$F(3,354)=16.04$, $p<.001$, $\eta_p^2=0.12$], but no main effect of condition [$F(1,118)=3.27$, $p=.07$, $\eta_p^2=0.03$], or interaction [$F(3,354)=1.29$, $p=.28$, $\eta_p^2=0.01$]. This suggests that participants attended to the four quadrants differently during the inference phase, suggesting that they made specific predictions about where the target would appear. There was no significant difference in RT across different trial types when the knowledge was explicitly told vs. when the knowledge was implicitly learned. Post-hoc Tukey HSD tests showed that RT in the impossible trials was reliably slower than that in the other three types of trials [p's<.03], the RT in the 2-quadrant disjunction trials was not reliably different from that in the 4-quadrant disjunction trials [p=.99], and the RT in the conjunction trials was reliably faster than both the 2-quadrant and 4-quadrant disjunction trials [p's<.01]. We then performed planned contrast analysis separately for the implicit and explicit conditions. The 2-quadrant and 4-quadrant disjunction trials were combined as one category in the analysis. For both conditions, RT in conjunction trials was significantly faster than that in disjunction trials, which in turn was faster than that in the impossible trials [p's<.01].

Additionally, we examined RT performance separately for learners (whose test phase accuracy>0.25, N=42) and non-learners (whose test phase accuracy⩽0.25, N=18). For learners, RT in conjunction trials was significantly faster than that in disjunction trials [p=.014], which in turn was faster than that in the impossible trials [p<.001]. For non-learners, RT in conjunction trials was marginally faster than that in disjunction trials [p=.09], but there was no difference in RT for the disjunction and impossible trials [p=.97]. This suggests that participants with higher test phase accuracy showed the effect more robustly than participants with lower test phase accuracy did.

These results suggest that when two objects each predicting a different outcome were presented at the same time, participants automatically made a conjunctive prediction which contained the shared property of the different outcomes.

Experiment 2

One explanation for faster RT in the conjunction trials in Experiment 1 was that the conjunctive quadrant was smaller in terms of spatial scope than the disjunctive quadrants. The smaller spatial scope might have facilitated visual search, leading participants to prioritize the conjunctive quadrant over the other quadrants. To examine this possibility, in Experiment 2, we aimed to equate the spatial scope of conjunctive and disjunctive quadrants in the inference phase.

Participants

A new group of 120 students (95 female, mean age=20.2 years, SD=1.9) from UBC participated for course credit.

Stimuli and Procedure

The stimuli and procedure in the experiment were the same as those in Experiment 1, except for one critical difference: During the exposure phase, after a color circle, the target could appear in three of the four quadrants. This means that in the inference phase, for each pair of color circles, two of the quadrants on the array would be consistent with a conjunctive inference, and the other two quadrants would be consistent with a disjunctive inference (Figure 4).

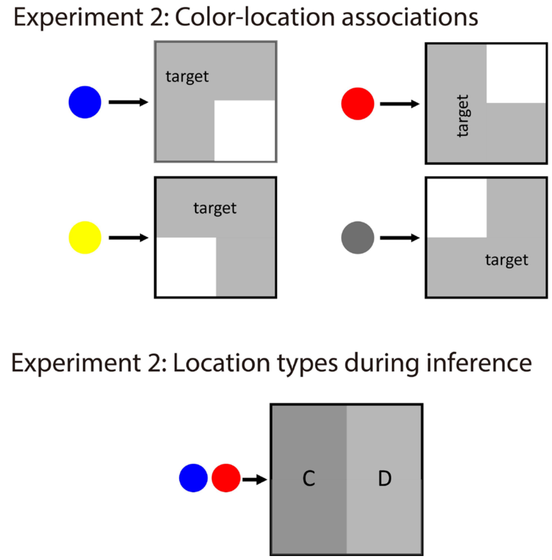

Figure 4: Experiment 2 paradigm. The stimuli and procedure were the same as those in Experiment 1, except that each color predicted the target would appear in three quadrants in the array during exposure. Consequently, two quadrants during the inference phase were consistent with a conjunctive inference (C), and the other two were consistent with a disjunctive inference (D).

Results and Discussion

The test phase accuracy for participants in the implicit condition was 0.31, which was not reliably above chance [p=0.11], suggesting that participants in the implicit condition did not successfully learn the color-location associations during exposure. This may be due to the difficulty of learning that the target could appear in three quadrants instead of two.

A 2 (condition: explicit vs. implicit, between-subjects) × 2 (trial type: conjunctive vs. disjunctive, within-subjects) mixed-design ANOVA revealed a marginal interaction between condition and trial type [$F(1,118)=3.865$, p=.05, $\eta_p^2=0.03$], but no main effect of condition [$F(1,118)=0.40$, p=.53, $\eta_p^2=0.00$], or trial type [$F(1,118)=1.22$, p=.27, $\eta_p^2=0.01$]. We then compared the RT in conjunction and disjunction trials separately for the implicit and explicit conditions. In the explicit condition, RT in conjunction trials was reliably faster than that in disjunction trials [$t(1,59)=2.03$, p<.05, d=0.24], but in the implicit condition, the RT in conjunction trials was not reliably different from that in disjunction trials [$t(1,59)=0.66$, p=.51, d=0.07] (see Figure 5).

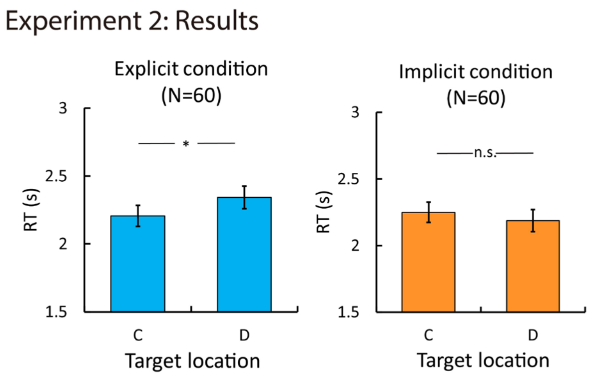

Figure 5: Experiment 2 results. The RT for each type of trials (C for conjunction, D for disjunction) was graphed separately for the explicit and implicit conditions (Error bar reflect ± 1 SE; $*p<.05$).

These results suggested that when participants learned the color-location associations, they automatically made conjunctive inferences when they saw two color circles, even when the conjunctive quadrants were of the same spatial scope as the disjunctive quadrants. On the other hand, if participants did not successfully learn the color-location associations, they failed to make such conjunctive inferences.

Experiment 3

In Experiments 1 and 2, the two color circles were presented simultaneously side by side during the inference phase to elicit conjunctive predictions. An alternative method to represent conjunctions is to combine two features into one object, such as combing the color red and the shape square into a red square (Treisman & Gelade, 1980; Singer & Gray, 1995). Therefore, in this experiment, we tested this alternative presentation where the two predictors were combined into a new object, rather than manifesting them as two different objects, to elicit conjunctive predictions.

Participants

A new group of 60 students (47 female, mean age=19.6 years, SD=2.6) from UBC participated for course credit. In the current experiment, only the implicit condition was examined (we did not examine the explicit condition due to time constraints in participant recruitment).

Stimuli and Procedure

The stimuli and procedure in the experiment were the same as those in Experiment 1, except for two critical differences.

First, during the exposure phase there were two color circles (red and blue, as described in Experiment 1) and two textured circles (dotted and stripy circles, see Figure 6). The two color circles were always presented with a filled texture, and the two textured circles were always presented in a black color (R/G/B: 0/0/0). The two color circles always predicted two parallel halves of the array (e.g., the top and bottom halves), and the two textured circles predicted the other two halves of the array (e.g., the left and right halves). The assignment of a color or texture to a given half was randomized across participants, but remained constant for a given participant throughout the experiment.

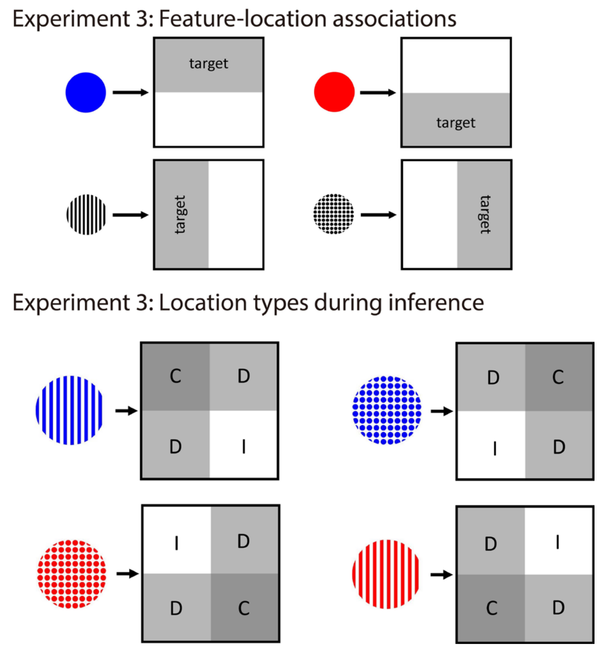

Figure 6. Experiment 3 paradigm. Two unique colors and two unique textures each predicted the target location in the following array during exposure. Circles with both a unique color and a unique texture were presented one at a time during inference. The trial types during the inference phase were the same as those in Experiment 1.

Second, during the inference phase participants saw one circle at a time on the screen. Each circle contained one of the two colors and one of the two textures presented in the exposure phase (i.e., a blue stripy circle, a blue dotted circle, a red stripy circle, or a red dotted circle). There were four trials for each unique colored textured circle. Since a color and a texture never predicted two parallel halves during exposure, there were three types of trials in the inference phase as in Experiment 1: conjunctive trials where the target could appear in a conjunctive quadrant (C), disjunctive trials where the target could appear in a disjunctive quadrant (D), and impossible trials where the target never appeared in a quadrant based on exposure (I).

Results and Discussion

The test phase accuracy in this experiment was 0.33, which was marginally above chance [p=.07], suggesting that learning was weak.

As before, we analyzed RT of correct trials in the inference phase (Figure 7). A one-way repeated-measures ANOVA revealed a main effect of trial type [$F(2,118)=5.32$, $p<.001$, $\eta_p^2=0.24$]. Post-hoc Tukey HSD tests showed that there was reliable RT difference in the conjunction trials and impossible trials [$p<.01$]. Other pair-wise comparisons were numerically similar to those in Experiment 1, but not statistically reliable [p's>.11]. These results suggest that the participants made conjunctive predictions when the two features were presented in a new object. However, the effect was not as strong as in previous experiments.

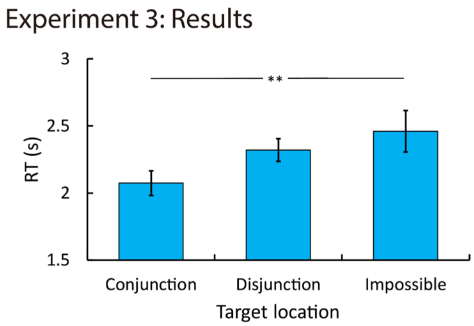

Figure 7: Experiment 3 results. The RT for each type of trials was graphed (Error bar reflect ± 1 SE; ***p*<.01).

General Discussion

In this study, we examined how predictions were made in the presence of two objects that were associated with two different outcomes. Using a visual search paradigm, unique colors (all three experiments) or textures (Experiment 3) predicted a specific location of the target in the search array in the exposure phase. In the inference phase, we examined where the target was expected to appear when two color circles (Experiments 1 and 2) or a circle with a unique color and a unique texture (Experiment 3) were presented at the same time. Importantly in the inference phase, the target appeared in any location with equal probability.

Based on the speed of visual search (RT), we found that participants were faster to find the target when it appeared in a conjunctive quadrant than in disjunctive or impossible quadrants. This was surprising because the simultaneous presentation of the two circles or features did not necessarily dictate a conjunctive or disjunctive inference. For example, just because the blue circle previously predicted the top half and the red circle previously predicted the left half, the blue and red circles together, in principle, could predict either the top left quadrant (conjunctive inference), or the top left, top right, and bottom left quadrants (disjunctive inference). What we found was that participants automatically prioritized the conjunctive quadrant over the disjunctive quadrant in the visual search task, at the presence of the two predictors. This conjunctive preference occurred without prior instructions, or even explicit awareness of the color-location associations.

Across all three experiments, participants were not told anything about where to look when two color circles or two different features were presented together. Therefore, the differential RT in the conjunctive quadrant indicated an automatic expectation resulting from the previously learned color- or feature-location associations during exposure.

In Experiment 1, the expectation to find the target in a location consistent with a conjunctive prediction was equally strong whether participants implicitly learned the associations or were explicitly told about the associations. However, in Experiment 2 when there was no successful learning of the associations in the implicit condition, this conjunctive prediction was absent. In fact, the conjunctive prediction was only present when participants were explicitly told about the color-location associations in the explicit condition. This suggests that the conjunctive predictions were only made when participants have successfully learned the color-location associations, either after implicit statistical learning, or after explicit instructions of these associations.

It is important to note that the disjunctive quadrants in the current study were exclusively disjunctive, not containing the conjunctive quadrant. The fact that the RT in the disjunction trials was faster than that in the impossible trials but slower than that in the conjunction trials suggests that the impossible quadrant may be inhibited and the conjunctive quadrant may be prioritized during visual search.

We think that both the learning process and the prediction process were implicit. In all three experiments, participants were not told anything about the object-location associations before the exposure phase in the implicit condition. That is, participants were only told to find the target in the search array and were not told that the object before each search array predicted the location of the target. Therefore, learning of the associations in the implicit condition was automatic and implicit. In the inference phase, there was no explicit instruction as to what to do with the two objects. Again, participants were only told to find the target in the search array. Moreover, the target in the inference phase could appear in any quadrant with equal probability, so the two objects were completely task-irrelevant. Finally, the RT was relatively fast so any explicit reasoning process may not occur in the period between object presentation and target search. For these reasons, we think that the conjunctive predictions were implicit.

There are several limitations of the current study. First, we only presented two objects side by side, or two features in a single object as cues. There might be other ways to represent such joint cues using semantic categories (e.g., if object A is associated with the "dog" category and object B is associated with the "small" category, will people automatically predict Chihuahuas and Pomeranians upon seeing A and B?). Second, we only used RT as a measure to probe whether participants made conjunctive or disjunctive predictions. A richer method can involve eye tracking to see the timecourse of attention to the different quadrants in the inference phase. Finally, there was a confound of proximity in the current study, where the conjunctive quadrant was spatially closer to the disjunctive quadrant than to the impossible quadrant. This could explain the RT advantage of the disjunction trials over the impossible trials.

In conclusion, the current results suggest that in the presence of multiple predictors, statistical learning generates automatic expectations about the outcomes in a conjunctive fashion.

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